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A METHOD OF CONSTRUCTING PROGRAMMED MOTIONS†

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An algorithm is proposed for constructing a control function for which the solution of a non-linear system of differential equations will go from its initial state to an arbitrarily small neighbourhood of a given final state. The problem of interorbital flight is considered.

Algorithms have been obtained [1, 2] for constructing control functions for which the solutions of linear and quasi-linear systems of differential equations will satisfy given boundary conditions. In this paper we investigate an analogous type of boundary-value problem for non-linear controllable systems in a bounded domain of phase space.

1. FORMULATION OF THE PROBLEM

The object of our investigation is the system

$$\dot{x} = f(x, u) \tag{1.1}$$

where

$$x = (x^1, \dots, x^n)^*, \quad x \in R^n; \quad u = (u^1, \dots, u^r)^*, \quad u \in R^r, \quad r \leqslant n \tag{1.2}$$
$$t \in [0, 1]; \quad f \in C^3(R^n \times R^r; \ R^n), \quad f = (f_1, \dots, f_n)^*$$

$$\|x\| < C_1, \quad \|u\| < C_2 \tag{1.3}$$

Suppose we are given the following states:

$$x(0) = 0, \quad x(1) = x_1; \quad x_1 = (x_1^1, \dots, x_1^n)^*, \quad \|x_1\| < C_1 \tag{1.4}$$

Problem. It is required to find functions $x(t) \in C^1[0, 1)$; $u(t) \in C^1[0, 1)$ which satisfy system (1.1) and conditions (1.3) such that the following relations are satisfied:

$$x(0) = 0, \quad x(t) \to x_1 \quad \text{as} \quad t \to 1 \tag{1.5}$$

We will call the pair $x(t)$, $u(t)$ a programmed motion.

2. SOLUTION OF THE PROBLEM

Let $u_1 \in R^r$; $u_1 = (u_1^1, \dots, u_1^r)$ be a vector in the domain (1.3) satisfying the conditions

$$f(x_1, u_1) = 0 \tag{2.1}$$

Using (1.2), we write system (1.1) in the form

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$$\dot{x}^i = \sum_{j=1}^{n} \frac{\partial f^i}{\partial x^j}(x_1, u_1)(x^j - x_1^j) + \sum_{j=1}^{r} \frac{\partial f^i}{\partial u^j}(x_1, u_1)(u^j - u_1^j) + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\partial^2 f^i}{\partial x^j \partial x^k} \times$$
$$\times (\tilde{x}, \tilde{u})(x^j - x_1^j)(x^k - x_1^k) + \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{r} \frac{\partial^2 f^i}{\partial x^k \partial u^j}(\tilde{x}, \tilde{u})(x^k - x_1^k)(u^j - u_1^j) +$$
$$+ \frac{1}{2}\sum_{j=1}^{r}\sum_{k=1}^{r} \frac{\partial^2 f^i}{\partial u^j \partial u^k}(\tilde{x}, \tilde{u})(u^j - u_1^j)(u^k - u_1^k) \tag{2.2}$$

where

$$\tilde{x} = x_1 + \theta_i(x - x_1), \quad \tilde{u} = u_1 + \theta_i(u - u_1); \quad \theta_i \in (0, 1)$$
$$\|\tilde{x}\| < C_1, \quad \|\tilde{u}\| < C_2 \tag{2.3}$$

We will seek a solution of the problem in the form

$$x^i(t) = a^i(t)(1 - t) + x_1^i, \quad i = 1, \ldots, n \tag{2.4}$$

$$u^j(t) = b^j(t)(1 - t) + u_1^j, \quad j = 1, \ldots, r \tag{2.5}$$

Substituting (2.4) and (2.5) into system (2.2), we obtain a system which may be expressed in vector notation as follows:

$$(1 - t)\dot{a} = a + (1 - t)Pa + (1 - t)Qb + R(a, b, t) \tag{2.6}$$
$$P = \{P_j^i\}, \quad i, j = 1, \ldots, n; \quad R = (R^1, \ldots, R^n)^*$$
$$Q = \{q_j^i\}, \quad i = 1, \ldots, n; \quad j = 1, \ldots, r$$

Conditions (1.3), (1.4), (2.4) and (2.5) give

$$\|a(t)(1 - t) + x_1\| < C_1, \quad \|b(t)(1 - t) + u_1\| < C_2$$
$$t \in [0, 1]; \quad a(0) = -x_1 \tag{2.7}$$

We change the variable t by the formula

$$1 - t = e^{-\alpha\tau}, \quad \tau \in [0, +\infty) \tag{2.8}$$

where $\alpha > 0$ is an as yet undetermined constant. Then system (2.6) and conditions (2.7) become

$$\frac{d\bar{a}}{d\tau} = \alpha\bar{a} + \alpha P e^{-\alpha\tau}\bar{a} + \alpha Q e^{-\alpha\tau}\bar{b} + \alpha R(\bar{a}, \bar{b}, \tau); \quad \tau \in [0, +\infty) \tag{2.9}$$

$$\left\|\bar{a}(\tau)e^{-\alpha\tau} + x_1\right\| < C_1, \quad \left\|\bar{b}(\tau)e^{-\alpha\tau} + u_1\right\| < C_2 \tag{2.10}$$
$$\bar{a}(0) = -x_1; \quad \tau \in [0, +\infty); \quad \bar{a}(\tau) = a(t(\tau)), \quad \bar{b}(\tau) = b(t(\tau))$$

We introduce variables $c(\tau)$ and $d(\tau)$ by the relations

$$\bar{a}(\tau) = c(\tau)e^{\alpha\tau}, \quad \bar{b}(\tau) = d(\tau)e^{\alpha\tau}; \quad \tau \in [0, +\infty) \tag{2.11}$$

Substituting expressions (2.11) into system (2.9) and taking conditions (2.10) into consideration, we have

$$\frac{dc}{d\tau} = \alpha e^{-\alpha\tau} Pc + \alpha e^{-\alpha\tau} Qd + R(c, d)e^{-\alpha\tau}, \quad \tau \in [0, +\infty) \tag{2.12}$$

$$\|c(\tau)+x_1\|<C_1,\ \ \|d(\tau)+u_1\|<C_2;\ \ \tau\in[0,\ +\infty),\ \ c(0)=-x_1 \tag{2.13}$$

Along with (2.12), we consider the system

$$\frac{dc}{d\tau}=\alpha e^{-\alpha\tau}Pc+\alpha e^{-\alpha\tau}Qd,\ \ \tau\in[0,\ +\infty) \tag{2.14}$$

We will seek a vector function $d(\tau)$ that will guarantee exponential stability of system (2.14).

Let $\bar{q}_i$ (throughout this section, $i = 1, \dots, r$) denote the ith column of the matrix Q. We construct the following matrix

$$S=\{\bar{q}_1,\ \dots,\ P^{k_1-1}\bar{q}_1,\ \dots,\ \bar{q}_r,\ \dots,\ P^{k_r-1}\bar{q}_r\} \tag{2.15}$$

where k_i is the maximum number of columns of the form $\bar{q}_i, P\bar{q}_i, \dots, P^{k_i-1}\bar{q}_i$ such that the vectors $\bar{q}_1, P\bar{q}_1, \dots, P^{k_1-1}\bar{q}_1, \dots, \bar{q}_r, \dots, P^{k_r-1}\bar{q}_r$ are linearly independent. If the rank of matrix (2.15) is n, then the transformation

$$c=Sy \tag{2.16}$$

reduces system (2.14) to the form

$$\frac{dy}{d\tau}=\alpha S^{-1}PSe^{-\alpha\tau}y+\alpha S^{-1}Qe^{-\alpha\tau}d \tag{2.17}$$

The matrices $S^{-1}PS$ and $S^{-1}Q$ have the form [1]

$$S^{-1}PS=\{\bar{e}_2,\ \bar{e}_3,\ \dots,\ \bar{e}_{k_1},\ \bar{g}_{k_1},\ \dots,\ \bar{e}_{k_{r-1}+2},\ \dots,\ \bar{e}_{k_r},\ \bar{g}_{k_r}\} \tag{2.18}$$

$$\bar{e}_i=(0,\ \dots,\ 1,\ \dots,\ 0)^*_{n\times 1}\ \ (1 - \text{in the } i\text{th place})$$

$$\bar{g}_{k_i}=(-g^0_{k_1},\ \dots,\ -g^{k_1-1}_{k_1},\ \dots,\ -g^0_{k_i},\ \dots,\ -g^{k_i-1}_{k_i},\ 0,\ \dots,\ 0)^*_{n\times 1}$$

$$P^{k_i}\bar{q}_i=-\sum_{j=0}^{k_1-1}g^j_{k_1}P^j\bar{q}_1-\dots-\sum_{j=0}^{k_i-1}g^j_{k_i}P^j\bar{q}_i \tag{2.19}$$

$$S^{-1}Q=\{\bar{e}_1,\ \dots,\ \bar{e}_{k_i+1},\ \dots,\ \bar{e}_{\gamma+1}\};\ \ \gamma=k_1+\dots+k_{r-1} \tag{2.20}$$

The constants $g^j_{k_1}$ $(j = 0, \dots, k_1-1), \dots, g^j_{k_i}$ $(j = 0, \dots, k_i-1)$ in (2.19) are the coefficients of the expansion of the vector $P^{k_i}\bar{q}_i$ in terms of the vectors $P^j\bar{q}_1 (j = 0, \dots, k_1-1), \dots, P^j\bar{q}_i (j = 0, \dots, k_i-1)$.

Consider the problem of stabilizing a system of the form

$$\frac{dy_{k_i}}{d\tau}=\{\bar{e}^{k_i}_2,\ \dots,\ \bar{e}^{k_i}_{k_i},\ \bar{\bar{g}}_{k_i}\}\alpha e^{-\alpha\tau}y_{k_i}+\bar{e}^{k_i}_i\alpha e^{-\alpha\tau}d^i \tag{2.21}$$

where

$$y_{k_i}=(y^1_{k_i},\ \dots,\ y^{k_i}_{k_i})^*_{k_i\times 1}$$

$$\bar{e}^{k_i}_i=(0,\ \dots,\ 1\ \dots,\ 0)^*_{k_i\times 1}\ \ (1 - \text{in the } i\text{th place})$$

$$\bar{\bar{g}}_{k_i}=(-g^0_{k_i},\ \dots,\ -g^{k_i-1}_{k_i})^*_{k_i\times 1},$$

$$d=(d^1,\ \dots,\ d^r)^*$$

In scalar form, system (2.21) may be written as

$$\frac{dy_{k_i}^1}{d\tau} = -\alpha g_{k_i}^0 e^{-\alpha\tau} y_{k_i}^{k_i} + \alpha e^{-\alpha\tau} d^i$$
$$\frac{dy_{k_i}^2}{d\tau} = \alpha e^{-\alpha\tau} y_{k_i}^1 - \alpha g_{k_i}^1 e^{-\alpha\tau} y_{k_i}^{k_i}$$
$$\dots$$
$$\frac{dy_{k_i}^{k_i-1}}{d\tau} = \alpha e^{-\alpha\tau} y_{k_i}^{k_i-2} - \alpha g_{k_i}^{k_i-2} e^{-\alpha\tau} y_{k_i}^{k_i} \tag{2.22}$$
$$\frac{dy_{k_i}^{k_i}}{d\tau} = \alpha e^{-\alpha\tau} y_{k_i}^{k_i-1} - \alpha g_{k_i}^{k_i-1} e^{-\alpha\tau} y_{k_i}^{k_i}$$

Let $y_{k_i}^{k_i} = \alpha^{k_i}\psi$. Using the last equation of system (2.22) and induction, we have

$$y_{k_i}^{k_i} = \alpha^{k_i}\psi$$
$$y_{k_i}^{k_i-1} = \alpha^{k_i-1} e^{\alpha\tau}\psi^{(1)} + g_{k_i}^{k_i-1}\alpha^{k_i}\psi$$
$$y_{k_i}^{k_i-2} = \alpha^{k_i-2} e^{2\alpha\tau}\psi^{(2)} + (\alpha^{k_i-1} e^{2\alpha\tau} + \alpha^{k_i-1} e^{\alpha\tau} g_{k_i}^{k_i-1})\psi^{(1)} + g_{k_i}^{k_i-2}\alpha^{k_i}\psi$$
$$\dots \tag{2.23}$$
$$y_{k_i}^1 = \alpha e^{(k_i-1)\alpha\tau}\psi^{(k_i-1)} + r_{k_i-2}(\tau)\psi^{(k_i-2)} + \dots + r_1(\tau)\psi^{(1)} + \alpha^{k_i} g_{k_i}^1 \psi$$

Differentiating the last equality of (2.23), we obtain from the first equation of system (2.22)

$$\psi^{(k_i)} + \varepsilon_{k_i-1}(\tau)\psi^{(k_i-1)} \dots + \varepsilon_0(\tau)\psi = e^{-k_i\alpha\tau} d^i \tag{2.24}$$

The functions $r_{k_i-2}(\tau), \dots, r_1(\tau)$ in (2.23) are linear combinations of exponential functions with exponents not exceeding $(k_i - 1)\alpha\tau$. The functions $\varepsilon_{k_i-1}(\tau), \dots, \varepsilon_0(\tau)$ in (2.24) are linear combinations of exponential functions with non-positive exponents.

Let

$$\upsilon^i = e^{-k_i\alpha\tau} d^i \tag{2.25}$$

We put

$$\upsilon^i = \sum_{j=1}^{k_i} (\varepsilon_{k_i-j}(\tau) - \gamma_{k_i-j})\psi^{(k_i-j)} \tag{2.26}$$

where $\gamma_{k_i-j} (j = 1, \dots, k_i)$ are chosen so that the roots $\lambda_{k_i}^1, \dots, \lambda_{k_i}^{k_i}$ of the equation

$$\lambda^{k_i} + \gamma_{k_i-1}\lambda^{k_i-1} + \dots + \gamma_0 = 0$$

satisfy the conditions

$$\lambda_{k_i}^i \neq \lambda_{k_i}^j, \quad i \neq j; \quad \lambda_{k_i}^j < -(2k_i + 1)\alpha - 1; \quad j = 1, \dots, k_i \tag{2.27}$$

Using relations (2.16), (2.22), (2.25) and (2.26), we obtain

$$d^i = e^{k_i\alpha\tau}\delta_{k_i} T_{k_i}^{-1} S_{k_i}^{-1} c \tag{2.28}$$

where

$$\delta_{k_i} = (\varepsilon_{k_i-1}(\tau) - \gamma_{k_i-1}, \dots, \varepsilon_0(\tau) - \gamma_0)$$

T_{k_i} is the matrix of system (2.2), that is, $\overline{\psi} = (\psi^{(k_i-1)}, \ldots, \psi)^*$; $y_{k_i} = T_{k_i}\overline{\psi}$; $S_{k_i}^{-1}$ is the matrix consisting of the corresponding k_i-rows of the matrix S^{-1}.

Substituting expressions (2.28) into the right-hand side of system (2.14), we conclude that its solution $c(\tau)$ with initial data

$$c(0) = -x_1 \tag{2.29}$$

has the limit

$$\|c(\tau)\| \leqslant M_0\|x_1\|e^{-\lambda\tau}, \quad \lambda > 1 \tag{2.30}$$

Consider system (2.12) closed by the control (2.28), assuming in addition that its solutions satisfy initial condition (2.29) and constraints (2.13). It can be represented in the form

$$dc/d\tau = A(\tau)c + g(c, \tau) \tag{2.31}$$

where

$$A(\tau) = \alpha e^{-\alpha\tau}P + \alpha e^{-\alpha\tau}Qe^{k\alpha\tau}\delta_k T_k^{-1}S_k^{-1}$$

$$e^{k\alpha\tau}\delta_k T_k^{-1}S_k^{-1} = (e^{k_1\alpha\tau}\delta_{k_1}T_{k_1}^{-1}S_{k_1}^{-1}, \ \ldots, \ e^{k_r\alpha\tau}\delta_{k_r}T_{k_r}^{-1}S_{k_r}^{-1})^*$$

$$g(c, \tau) = e^{-\alpha\tau}R(c, d)$$

Conditions (1.2), (2.2), (2.13), (2.3) and (2.28) guarantee the existence of constants $L > 0$ and $M > 0$ such that

$$\|g(c, \tau)\| \leqslant Le^{M\alpha\tau}\|c\|^2; \quad M > 2k_i \tag{2.32}$$

In addition, it follows from (2.27) and (2.30) that the system

$$dc/d\tau = A(\tau)c \tag{2.33}$$

is exponentially stable.

We make the change of variables in (2.31)

$$c(\tau) = z(\tau)e^{-M\alpha\tau} \tag{2.34}$$

As a result, we obtain

$$dz/d\tau = B(\tau)z + g_1(z, \tau) \tag{2.35}$$

$$z(0) = -x_1 \tag{2.36}$$

$$B(\tau) = A(\tau) + M\alpha E; \quad g_1(z, \tau) = e^{M\alpha\tau}g(ze^{-M\alpha\tau}, \tau)$$

where E is the identity matrix. Using relations (2.32) and (2.34), we have

$$\|g_1(z, \tau)\| \leqslant L\|z\|^2 \tag{2.37}$$

Obviously, for sufficiently small $\alpha > 0$, the exponential stability of system (2.33) implies that of the system

$$dz/d\tau = B(\tau)z \tag{2.38}$$

with exponent $-\beta = -\lambda + \alpha M$, where $-\lambda$ is the exponent of exponential stability of system (2.33).

Let $\Phi(\tau)$, $\Phi(0) = E$ be the fundamental matrix of system (2.38). The solution of system (2.35) with initial data (2.36) which remains in the domain

$$\left\|z(\tau)e^{-M\alpha\tau} + x_1\right\| < C_1, \quad \tau \in [0, +\infty) \tag{2.39}$$

takes the form

$$z(\tau) = -\Phi(\tau)x_1 + \int_0^\tau \Phi(\tau)\Phi^{-1}(t)g_1(z,\ t)dt \tag{2.40}$$

By the limit (2.30), we obtain

$$\|\Phi(\tau)\| \leq Ke^{-\beta\tau} \tag{2.41}$$

where K is a constant which generally depends on β.
Let us replace inequality (2.39) by a stronger one

$$\|z(\tau)\| < C_1 - \|x_1\|; \quad \tau \in [0,\ \infty) \tag{2.42}$$

It follows from relations (2.37) and (2.40) that

$$\|z(\tau)\| \leq Ke^{-\beta\tau}\|x_1\| + \int_0^\tau e^{-\beta(\tau-t)}K\Delta\|z(t)\|dt \tag{2.43}$$

$$\Delta = L(C_1 - \|x_1\|)$$

whence, from known results [3], we have

$$\|z(\tau)\| \leq Ke^{-\mu\tau}\|x_1\|, \quad \mu = \beta - K\Delta \tag{2.44}$$

Let us assume that

$$\mu = \beta - K\Delta > 0 \tag{2.45}$$

Suppose x_1 and u_1 satisfy that conditions

$$(K+1)\|x_1\| < C_1 \tag{2.46}$$

$$\|\delta_k(0)T_k^{-1}(0)S_k^{-1}\|K\|x_1\| + \|u_1\| < C_2$$

If we substitute the functions (2.40) into formulae (2.34), (2.28), (2.11), (2.4) and (2.5), then, by the derivation of Eqs (1.2), (2.6), (2.9) and (2.12), whose legitimacy is guaranteed by conditions (2.46), (2.42), (2.39), (2.13), (2.10), (2.7) and (2.3), we obtain the solution of the problem in question.
On the basis of these arguments, the following theorem holds.

Theorem. Let C_1, C_2 and α be numbers, x_1 and u_1 vectors and K a constant (defined by quantities $\lambda_{k_i}^j (i = 1, \ldots, r; j = 1, \ldots, k_i)$ satisfying inequalities (2.27)) for which conditions (2.1), (2.45) and (2.46) hold, and suppose moreover that matrix (2.15) is non-singular. Then a solution of the problem formulated above exists which reduces to solving the stabilization problem for a linear stationary system, integrating system (2.35), (2.36) and then returning to the original variables t, x by using formulae (2.34), (2.11), (2.12), (2.5) and (2.4).

3. SOLUTION OF THE PROBLEM OF INTERORBITAL FLIGHT

As an illustration of the proposed method, we present the solution of the problem of steering a point mass of variable mass $m(t)$, moving in a circular orbit of radius r_0 about a mass M with constant angular velocity α_0 in a central gravitational field, to a given point in the orbital plane. As control we choose the reactive force. The system of equations in deviations relative to the above motion in a circular orbit is [4]

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \nu_1(x_1,\ x_4) + u_1$$

$$\dot{x}_3 = x_4, \quad \dot{x}_4 = \nu_2(x_1,\ x_2,\ x_4) + \nu_3(x_1)u_2 \tag{3.1}$$

where

$$x_1 = r - r_0,\quad x_2 = \dot r,\quad x_3 = \psi - \alpha_0 t,\quad x_4 = \dot\psi - \alpha_0$$
$$u_1 = a_r \dot m / m,\quad u_2 = a_\psi \dot m / m$$
$$\nu_1 = -\frac{\nu}{(x_1 + r_0)^2} + (x_1 + r_0)(x_4 + \alpha_0)^2$$
$$\nu_2 = -2\frac{x_2(x_4 + \alpha_0)}{x_1 + r_0},\quad \nu_3 = \frac{1}{x_1 + r_0}$$

$\dot r$ is the radial velocity of the point, ψ is the polar angle, $\dot\psi$ is the rate of change of the polar angle, a_r and a_ψ are the projections of the relative velocity vector of the deviating particle on the direction of the radius and the transversal direction, respectively, and $\nu = \nu^0 M$, where ν^0 is the universal constant of gravitation. Conditions (1.3), (1.5) and (2.1) become

$$\|x\| < C_1,\quad x = (x_1, \ldots, x_4)^*;\quad \|u\| < C_2,\quad u = (u_1, u_2)^* \tag{3.2}$$
$$x(0) = 0;\ x(t) \to x^1 \ \text{as}\ t \to 1$$
$$x^1 = (x_1^1, x_2^1, x_3^1, x_4^1)^*,\quad u^1 = (u_1^1, u_2^1)^*$$
$$x_2^1 = 0,\quad x_4^1 = 0;\quad u_1^1 = -\nu_1(x_1^1),\quad u_2^1 = -\frac{\nu_2(x_1^1)}{\nu_3(x_1^1)} = 0 \tag{3.3}$$

The constraints (2.3) and the matrices P, Q and S on the right-hand side of system (2.12) may be written as follows:

$$\|x^1 + c\| < C_1,\quad \|u^1 + d\| < C_2,\quad c = (c_1, \ldots, c_4),\quad d = (d_1, d_2) \tag{3.4}$$
$$P = \begin{Vmatrix} 0 & 1 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & 0 & 0 \end{Vmatrix},\quad Q = \begin{Vmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & \beta_0 \end{Vmatrix},\quad S = \begin{Vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & \beta_0 a_{24} \\ 0 & 0 & 0 & \beta_0 \\ 0 & a_{42} & \beta_0 & 0 \end{Vmatrix}$$
$$a_{21} = \frac{\partial \nu_1}{\partial x_1}(x_1^1),\quad a_{24} = \frac{\partial \nu_1}{\partial x_4}(x_1^1),\quad a_{42} = \frac{\partial \nu_2}{\partial x_2}(x_1^1),\quad \beta_0 = \nu_3(x_1^1)$$

Obviously, $\det S \neq 0$ for all x_1^1, x_3^1. This implies that system (2.14) is stabilizable, irrespective of the choice of x_1^1, x_3^1.

After solving the stabilization problem for system (2.14), we use formula (2.28) to find functions d_1 and d_2 for which system (2.14), closed by them, is exponentially stable with exponent $-\lambda(\alpha) < 0$ for all $\alpha \in [0, +\infty)$. Estimating the mixed second partial derivatives of the right-hand sides of system (3.1) with respect to x_i ($i = 1, \ldots, 4$) and u_i ($i = 1, 2$), taking constraints (3.4) into consideration, and assuming that α is chosen in a bounded domain, we obtain the constants M and L. Solving the inequality $-\lambda(\alpha) + \alpha M < 0$, we find the constant

$$-\beta = -\lambda(\alpha_0) + \alpha_0 M < 0$$

After estimating the norm of the fundamental matrix of system (2.38) (this may be done using an estimate of the fundamental matrix of system (2.14), closed by the stabilizing controls), we obtain the constant K. We then choose x_1^1, x_3^1 so that conditions (2.45) and (2.46) are satisfied.

At the concluding step, we solve a Cauchy problem for system (2.31) with initial data $(-x_1^1, 0, -x_3^1, 0)$ and return to the original independent variable t by formula (2.8). As a result, we obtain a pair of functions

$$x(t) = (x_1(t), \ldots, x_4(t))^*,\quad u(t) = (u_1(t), u_2(t))^*$$

satisfying system (3.1) and conditions (3.2).

4. NUMERICAL MODELLING

In the process of numerical modelling, the following auxiliary system was integrated

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \nu_1(x_1 + x_1^1, x_4) + u_1 + u_1^1, \quad \dot{x}_3 = x_4, \quad \dot{x}_4 = \nu_2(x_1 + x_1^1, x_2, x_4) + \nu_3(x_1 + x_1^1)u_2$$

where

$$u_1^1 = \frac{\nu}{(x_1^1 + r_0)^2} - (x_1^1 + r_0)\alpha_0^2$$

$$\alpha_0 = \sqrt{\frac{\nu}{r_0^3}}\ \text{c}^{-1}, \quad x_1^1 = 100, \qquad r_0 = 7 \cdot 10^6\ \text{м}, \quad x_3^1 = \alpha_0 \cdot 10^{-6}$$

in the interval [0, 0.99] with initial data

$$x_1(0) = -x_1^1, \quad x_2(0) = 0, \quad x_3(0) = -x_3^1, \quad x_4(0) = 0$$

closed by controls

$$u_1 = -\frac{1}{\alpha^3 a_{42}}[a_{42}e^{2\alpha\tau}\alpha((\gamma_2 - 6)\alpha -$$

$$-(\gamma_1 - 11))x_1 - (\gamma_2 - 6)\alpha^2 a_{42}e^{\alpha\tau}x_2 + 6e^{3\alpha\tau}x_3]$$

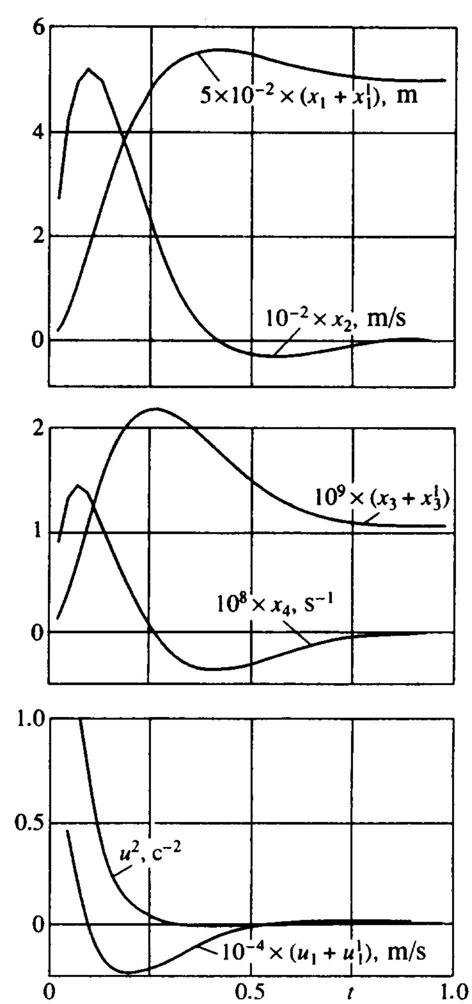

Fig. 1

$$u_2 = -\frac{4e^{\alpha\tau}}{\alpha\beta_0}(a_{42}x_1 - x_4)$$

$$\alpha = 1/4, \quad \gamma_2 = 3\alpha, \quad \gamma_1 = 2\alpha^2 - \alpha^2 e^{-2\alpha\tau}\gamma_{23}$$

$$\gamma_{23} = a_{21} + a_{24}a_{42}$$

Figure 1 shows graphs corresponding to the required functions of the phase coordinates $x_1(t), x_2(t), x_3(t), x_4(t)$, and the controls $u_1(t)$, $u_2(t)$, which are programmed motions for system (3.1).

A preliminary analysis of the results of the modelling process enables us to draw the following conclusions:

(1) the greatest energy resources demanded by the control are expended for $u_1(t)$ and they depend directly on x_1^1 and the time of the motion;

(2) the constant L is of the order of 10^{-6}, and the choice of the quantity α therefore presents no particular difficulty;

(3) the problem of interorbital flight is easily solved using personal computers of average capacity.

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